POLARISATION OF A DIRAC PARTICLE AND PROTON-NEUTRON SCATTERING

S. P. MISRA

MATHEMATICS DEPARTMENT, RAVENSHAW COLLEGE, CUTTACK–3

(Received for publication March, 13, 1958).

ABSTRACT. The purpose of this paper has been to consider the polarisation problem for nucleon-nucleon scattering in the pseudoscalar meson theory. Attempts have always been made to explain polarisation in case of scattering by expanding in terms of centre of mass angular momentum and considering the scattering problem with the help of phase shifts in the different angular momentum and spin states. This procedure often contains the error of keeping only a few phase shifts without any theoretical justification We have avoided this source of error by taking the usual S-matrix in the second and fourth orders. Using this direct method, it has been observed that polarisation in case of such a scattering, when initially the partciles are unpolarised, is zero, which is in contradiction to the experimental results

INTRODUCTION

It is well known that the nuclear forces are best explained, at least in a qualitative manner, with the help of psudoscalar meson theory. We wish to consider polarisation in case of nucleon-nucleon scattering in this theory.

Since the experimental discovery of polarisation in case of scattering, many attempts have been made to explain them with the help of different models or from field theory with approximate calculations (Bhatia, 1950 ; Verde, 1955 ; Lepore, 1950). The approach of these attempts are similar to that of Mott (1932) and makes use of the centre of mass angular momentum expansion of the incoming and outgoing waves, and the subsequent calculations of different angular momentum states phase shifts only a few of which can be retained. Thus, besides the usual inaccuracy of the perturbation approach for the meson theories, the inaccuracy of retaining only a few angular momentum states without any theoretical justification in the individual cases, is further added.

To avoid at least the second error, we have considered the scattering problem from the general scattering matrix in the second and fourth orders for the particular case of neutron-proton scattering. We assume the incident particles to be unpolarised and calculate the cross-section for an arbitrary direction of polarisation of the outgoing proton beam. The dependance of this cross-section on the state of polarisation of the proton will give us the amount of polarisation of this beam as a result of scattering, and by angular momentum conservation, we shall automatically obtain the polarisation of the neutron beam.

It is, however, found that the scattering cross-section does not depend on the state of polarisation of the outgoing proton, thus going against the experimental result that polarisation should be obtained in case of such a scattering.

It is to be noticed, however, that the scatterers in the case of experiments are not free particles, but are bound. This might be the source of error in taking as we have done, the wave functions to be all those of free particles.

RELATIVISTIC POLARISATION STATES OF A SINGLE DIRAC PARTICLE

The four spinor wave function of the Dirac particle of energy-momentum p will be taken in the positive energy states as

$$\psi(p) = k \begin{pmatrix} \psi_{Ip} \\ \psi_{IIp} \end{pmatrix}$$

where the small component $\psi_{IIp} = \dfrac{\vec{\sigma}\cdot\vec{p}}{E_p+M}\,\psi_{Ip}$ and $\psi_{Ip} = \begin{pmatrix} \cos\frac{\alpha}{2} \\ \sin\frac{\alpha}{2}\exp(i\beta) \end{pmatrix}$

The invariant normalisation $\bar{\psi}(p)\psi(p) = 1$ gives us $k = \sqrt{\dfrac{E_p+M}{2M}}$. We adopt the notation of Heitler (1954) as regards the γ-matrices the energy-momenta and the summation conventions. We define the anti-symmetric polarisation operator as

$$\sigma_{\mu\nu} = 1/2i(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$$

such that the Pauli spin matrices $\vec{\sigma}$ are given as $\vec{\sigma} = (\sigma_{23}, \sigma_{31}, \sigma_{12})$. The relativistically covariant polarisation tensor is defined as

$$P_{\mu\nu} = \bar{\psi}(p)\sigma_{\mu\nu}\,\psi(p)$$

for the particle represented by the 4-spinor $\psi(p)$. Hence

$$\vec{P} \equiv \bar{\psi}(p)\,\vec{\sigma}\,\psi(p)$$

$$= \frac{E_p}{M}\,\vec{P}_{NR} - \frac{(\vec{p}\cdot\vec{P}_{NR})}{M(E_p+M)}\,\vec{p} \qquad \ldots (1)$$

where

$$\vec{P}_{NR} = \psi^*_{Ip} \vec{\sigma} \psi_{Ip} = (\sin\alpha\cos\beta,\ \sin\alpha\sin\beta,\ \cos\alpha)$$

is the non-relativistic polarisation vector.

Again, $$\vec{\sigma}_{k4} = (\sigma_{14}, \sigma_{24}, \sigma_{34}) = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}$$

gives us

$$\vec{P}_{k4} = \frac{i}{M} \vec{p} \times \vec{P}_{NR}$$

or,

$$\vec{P}_{k0} = \frac{\vec{p}}{M} \times \vec{P}_{NR} \qquad \ldots \quad (2)$$

It is easily seen that

$$\tfrac{1}{2} P_{\mu\nu} P^{\mu\nu} = \vec{P} \cdot \vec{P} - \vec{P}_{k0} \cdot \vec{P}_{k0} = 1$$

by eqns. (1) and (2) and further simplification, as we should expect of the polarisation.

From the above-mentioned equations it is also clear that the strictly space part (1) of the polarisation depends on the momentum of the particle unless we quantise polarisation parallel to the direction of the momentum itself, in which case $\vec{P} = \vec{P}_{NR}$. In such a case, the strictly relativistic part (2) also vanishes altogether.

In the general case, however, both the parts of the anti-symmetric tensor $P_{\mu\nu}$ stay, and have to be taken into account while considering coupling of this polarisation with any other tensor for an invariant description of such problems. It is also to be seen, that, if we consider, say, only positive energy states, then such states as $\exp(ipx) \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ are impossible.

If we replace α by $\pi - \alpha$ and β by $\beta + \pi$, such that

$$\psi_{Ip} \text{ changes to } \begin{pmatrix} \sin\frac{\alpha}{2} \\ -\cos\frac{\alpha}{2}\exp(i\beta) \end{pmatrix}$$

then the new $\psi(p)$ thus obtained is orthogonal to the original one, and has just the opposite polarisation even in the relativistic limit.

SECOND AND FOURTH ORDER PERTURBATION CALCULATIONS

In the specific problem of proton-neutron scattering, we take p_1, p_2 and p_3 and p_4 as the four-momenta of the incident proton and neutron and the outgoing proton and neutron respectively. The interaction hamiltonian density is taken as

$$H_i(x) = ig\bar{\psi}(x)\gamma_5\,\tau_k\psi(x)\phi_k(x) \qquad \ldots \quad (3)$$

The meson and nucleon propagation functions are respectively as

$$K_M\,(x_1-x_2) = -\frac{i}{(2\pi)^4}\int\frac{\exp\{(ik(x_1-x_2)\}}{k^2+\mu^2}\,dk$$

and

$$K_N(x_1-x_2) = \frac{i}{(2\pi)^4}\int\frac{(i\gamma k-M)}{k^2+M^2}\quad\exp\{ik(x_1-x_2)\}dk.$$

In the above, μ and M are respectively the meson and nucleon masses. Throughout, natural units $c = \hbar = 1$ have been chosen.

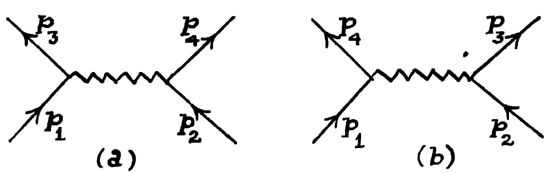

Fig. 1(a) & (b)

With summation over isotopic spin indices, the second order matrix elements become

$$S_2 = S_2^{(1)}+S_2^{(2)}, \qquad \ldots \quad (4)$$
$$S_2^{(1)} = ig^2(2\pi)^4\bar{\psi}(p_3)\gamma_5\,\psi(p_1)\bar{\psi}(p_4)\gamma_5\psi(p_2)[q^2+\mu^2]^{-1}$$
$$S_2^{(2)} = 2ig^2(2\pi)^4\psi(p_3)\gamma_5\,\psi(p_2)\bar{\psi}(p_4)\,\gamma_5\psi(p_1)[(q-p)^2+\mu^2]^{-1}$$

In the above result, $\psi(p_1)$ and $\bar{\psi}(p_3)$ are proton states and $\psi(p_2)$ and $\bar{\psi}(p_4)$ are neutron states, and $p = p_1-p_2$ and $q = p_1-p_3$.

For calculating the fourth order matrix element, we consider moderately high energies so that only the graphs figure 2 (a & b) and the graphs with p_1 and p_2 interchanged contribute the maximum.

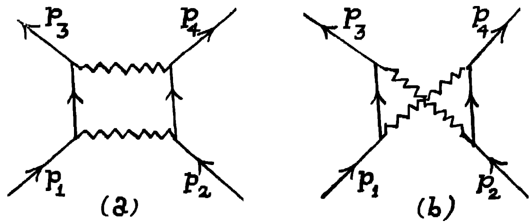

Fig. 2(a) & (b)

Then we easily obtain the respective scattering matrix elements as

$$S_4^{(a)} = g^4 \frac{1}{\lfloor 4} \cdot 4 \int d^4k \bar{\psi}(p_3)\gamma_5\tau_k\{i\gamma(p_1-k)-M\}\gamma_5\tau_l\psi(p_1)$$

$$\times \bar{\psi}(p_4)\gamma_5\tau_k\{i\gamma(p_2+k)-M\}\gamma_5\tau_l\psi(p_2)\times[k^2-2kp_1]^{-1}[k^2+2kp_2]^{-1}$$

$$[k^2+\mu^2)^{-1}[(q-k)^2+\mu^2]^{-1}$$

$$=5/6g^4\bar{\psi}(p_3)\gamma_\mu\psi(p_1)\bar{\psi}(p_4)\gamma_\nu\psi(p_2)I_{\mu\nu} \qquad \ldots \quad (5)$$

where

$$I_{\mu\nu} = \int \; d^4k \; k_\mu k_\nu[k^2+\mu^2]^{-1}[k^2-2kp_1]^{-1}[k^2+2kp_2]^{-1}\times[(q-k)^2+\mu^2]^{-1} \qquad \ldots (5a)$$

Similarly

$$S_4^{(b)} = -1/6g^4\bar{\psi}(p_3)\gamma_\mu\psi(p_1)\bar{\psi}(p_4)\gamma_\nu\psi(p_2)\; J_{\mu\nu} \qquad \ldots \quad (6)$$

where

$$J_{\mu\nu} = \int d^4k k_\mu \; k_\nu[k^2+\mu^2]^{-1}[k^2-2kp_1]^{-1}[k^2-2kp_4[(k-q)^2+\mu^2]^{-1} \quad \ldots \quad (6a)$$

There will be two other matrix elements with the exchange of p_1 and p_2 (or p_3 and p_4) given as

$$S_4^{(a')} = -2/3\, g^4\bar{\psi}(p_3)\gamma_\mu\psi(p_2) \;\; \bar{\psi}(p_4) \;\; \gamma_\nu\psi(p_1)\, I_{\mu\nu}' \qquad \ldots \quad (7)$$

where

$$I_{\mu\nu}' = I_{\mu\nu}(p_1 \Longleftrightarrow p_2) \qquad \ldots \quad (7a)$$

and

$$S_4^{(b')} = -\frac{2}{3}g^4\bar{\psi}(p_3)\gamma_\mu \; \psi(p_2)\bar{\psi}(p_4) \; \gamma_\nu \; \psi(p_1)J'_{\mu\nu} \qquad \ldots \quad (8)$$

where

$$J'_{\mu\nu} = J_{\mu\nu}(p_1 \Longleftrightarrow p_2) \qquad \ldots \quad (8a)$$

The numerical coefficients are seen to be different in the latter two cases from a consideration of the specific isotopic spin states of the neutrons and the protons and taking summation over such states for the intermediate nucleons.

Hence finally we obtain the scattering matrix up to fourth order as

$$\begin{aligned} S &= S_2 + S_4 \\ &= i[g^2 Q\bar{\psi}(p_3)\gamma_5\,\psi(p_1)\,\bar{\psi}(p_4)\,\gamma_5\,\psi(p_2) + g^2 Q'\bar{\psi}(p_3)\,\gamma_5\,\psi(p_2)\,\bar{\psi}(p_4)\gamma_5\,\psi(p_1) \\ &\quad + g^4 Q_{\mu\nu}\,\bar{\psi}(p_3)\gamma_\mu\,\psi(p_1)\,\bar{\psi}(p_4)\,\gamma_\nu\,\psi(p_2) + g^4 Q'_{\mu\nu}\,\bar{\psi}(p_3)\gamma_\mu\,\psi(p_2)\,\bar{\psi}(p_4)\gamma_\nu\,\psi(p_1)] \end{aligned} \quad \cdots (9)$$

where

$$Q = (2\pi)^4[q^2+\mu^2]^{-1}, \qquad Q' = 2(2\pi)^4[(q-p)^2+\mu^2]^{-1}$$

$$iQ_{\mu\nu} = \frac{5}{6}\,I_{\mu\nu} - \frac{1}{6}\,J_{\mu\nu} \quad \cdots (9a)$$

and

$$iQ'_{\mu\nu} = -\frac{2}{3}I'_{\mu\nu} - \frac{2}{3}J'_{\mu\nu}$$

For the evaluation of the respective $I_{\mu\nu}$ and $J_{\mu\nu}$ integrals, we make use of the representation.

$$1/(a\,b\,c\,d) = 6\int_0^1 dx \int_0^1 dy \int_0^1 dz\; x(1-x)[ay+b(1-y))x+(cz+d(1-z)(1-x)]^{-4}$$

and employ the wellknown techniques of Feynman to evaluate the resulting integrals.

Thus we shall have

$$I_{\mu\nu} = i\pi^2 \int_0^1 dx \int_0^1 dy \int_0^1 dz[D_1^{-2}\;P_{1\mu}(x,y,z)P_{1\nu}\;(x,y,z) + \tfrac{1}{2}\,\delta_{\mu\nu}\,D_1^{-1}] \quad \cdots (10)$$

where

$$P_{1\mu}\,(x,y,z) = p_{1\mu}\,x(1-y) - p_{2\mu}\,xy + q_\mu\,(1-x)z, \quad \cdots (10a)$$

$$\Delta = -\,(1-x)(\mu^2+q^2z)$$

and

$$\begin{aligned} D_1 &= -\,(P_1(x,y,z))^2 - \Delta \\ &= M^2x^2(1-2y)^2 + \mu^2(1-x) + q^2z(1-x) \\ &\qquad - p^2x^2y(1-y) - 2(pq)\,x\,(1-x)yz \end{aligned} \quad \cdots (10b)$$

The integrals $J_{\mu\nu}$, $I'_{\mu\nu}$ and $J_{\mu\nu}'$ can also be written down in a similar way and will have the same form as above. Hence by (9*a*), (10*a*) and (10), we have

$$\begin{aligned} Q_{\mu\nu} &= A\,\delta_{\mu\nu} + S_{\mu\nu} \\ Q'_{\mu\nu} &= A'\,\delta_{\mu\nu} + S'_{\mu\nu} \end{aligned} \quad \cdots (11)$$

where A and A' are real invariant functions of momenta and $S_{\mu\nu}$ and $S'_{\mu\nu}$ are the components of the sum of symmetric tensor products of the momentum-energy four vectors. Hence in (11), we have

$$S_{\mu\nu} = S_{\nu\mu}$$

and also S_{kl}, S'_{kl} $(k, l = 1, 2, 3)$ and S_{44}, S'_{44} are all real and all the remaining components of S and S' are pure imaginary.

Hence we have

$$\begin{aligned} Q^*_{kl} &= Q_{kl}, \qquad k, l = 1, 2, 3, \\ Q^*_{4k} &= -Q_{4k}, \end{aligned} \qquad \ldots \quad (12)$$

and the Q's as well are also symmetric.

We shall now adopt the properties (12) of $Q_{\mu\nu}$ and $Q'_{\mu\nu}$ and employ the expression (9).

In (9), we shall explicitly take $\psi(p_3)$ to be in the positive energy state with a definite polarisation given as

$$\psi(p_3) = \frac{1}{\sqrt{2M\,(E_{p_2}+M)}} \begin{bmatrix} E_{p_3}+M \\ \vec{\sigma}\cdot\vec{p}_3 \end{bmatrix} \psi_{Ip_3}, \qquad \ldots \quad (13)$$

$$\psi_{Ip_3} = \begin{bmatrix} \cos\frac{\alpha}{2} \\ \sin\frac{\alpha}{2}\,\exp(i\beta) \end{bmatrix}.$$

$\frac{1}{4} \sum_{p_1,p_2,p_3} |S|^2$ with the notation that $\sum_p$ indicates summation over the spin states of particles of momentum p, will give us the scattering cross-section when the outgoing proton has a definite state of polarisation given by (13) and the incident beams are unpolarised. In the calculation of $\sum_{p_1,p_2,p_4} |S|^2$, we shall have to employ the positive energy state projection operators $\Lambda_+(p_1)$, $\Lambda_+(p_2)$ and $\Lambda_+(p_4)$,

$$\text{where } \Lambda_+(p) = \frac{-i\gamma p + M}{2E_p}\,\beta$$

$$= \frac{1}{2E_p} \begin{bmatrix} E^+_p & \vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p} & E^-_p \end{bmatrix} \qquad \ldots \quad (14)$$

with the abbreviation $E^+_p = E_p + M$ and $E^-_p = E_p - M$.

$\sum_{p_1,p_2,p_4} |S|^2$ will give rise to sixteen terms in all, many of which will vanish. The different types of terms that will occur have been considered in the appendix.

When expressed in terms of the Pauli spin matrices, all of them will be of the form

$$\bar{\psi}(p_3)\begin{bmatrix} A_1+\vec{\sigma}\,.\,\vec{B}_1 & A_2+\vec{\sigma}\,.\,\vec{B}_2 \\ A_3+\vec{\sigma}\,.\,\vec{B}_3 & A_4+\vec{\sigma}\,.\,\vec{B}_4 \end{bmatrix}\psi(p_3) \qquad \ldots \quad (15)$$

which, by (13), will simplify to

$$\frac{1}{2ME^+_{p_3}}\psi^* I p_3\{(E_{p_3})^2A_1-|\vec{p}_3|^2A_4+E^+{p_3}(\vec{B}_2-\vec{B}_3).\,\vec{p}_3$$

$$+\vec{\sigma}\,.\,[E^+{p_3})^2\vec{B}_1+|\vec{p}_3|^2\vec{B}_4-2(\vec{p}_3\,.\,\vec{B}_4)\,\vec{p}_3$$

$$+E^+_{p_3}(A_2-A_3)\vec{p}_3+iE^+_{p_3}(\vec{B}_2+\vec{B}_3)\times\vec{p}_3]\}\psi I p_3 \qquad \ldots \quad (15a)$$

In order that the scattering cross-section may depend on the state of polarisation of $\psi(p_3)$, the sum of the terms arising from the square bracket in (15a) from all the individual terms should not vanish. Since the final sum of all the terms must be real, in terms like (15a) we must consider only the real parts of A_1, A_4, $\vec{B}_2$ and $\vec{B}_3$ for the scattering cross-section that does not depends on the spin states of $\psi(p_3)$ and to obtain the spin dependance of the cross-section, we must consider the real parts of A_2, A_3, $\vec{B}_1$ and $\vec{B}_4$ and the pure imaginary parts of $\vec{B}_2$ and $\vec{B}_3$. It will however be seen in the appendix that in any matrix

$$O=\begin{bmatrix} A_1+\vec{B}_1\,.\,\vec{\sigma} & A_2+\vec{B}_2\,.\,\vec{\sigma} \\ A_3+\vec{B}_3\,.\,\vec{\sigma} & A_4+\vec{B}_4\,.\,\vec{\sigma} \end{bmatrix}$$

considered in (15), A_2 and A_3, $\vec{B}_1$ and $\vec{B}_4$ are pure imaginary and $\vec{B}_2$ and $\vec{B}_3$ are real. This shows that there is no preferred orientation of the spin of the proton as a result of scattering.

APPENDIX

The type of terms that will occur in $\sum_{p_1,p_2,p_4}|S|^2$ are the following : (neglecting real coefficients which are unimportant for our purpose)

(1) $\bar{\psi}(p_3)\gamma_5\Lambda_+(p_1)\gamma_5\beta\psi(p_3)\ Sp[\beta\gamma_5\Lambda_+(p_2)\gamma_5\beta\Lambda_+(p_4)]$

(2) $\bar{\psi}(p_3)\gamma_5\Lambda_+(p_1)\gamma_5\beta\Lambda_+(p_4)\beta\gamma_5\Lambda_+(p_2)\gamma_5\beta\psi(p_3)$

(3) $\bar{\psi}(p_3)\gamma_\mu\Lambda_+(p_1)\beta\gamma_\lambda\psi(p_3)Q_{\mu\nu}Q_{\lambda k}\ Sp\,[\beta\gamma_\nu\Lambda_+(p_2)\beta\gamma_k\Lambda_+(p_4)]$

(4) $\bar{\psi}(p_3)\gamma_5\Lambda_+(p_1)\beta\gamma_\mu\psi(p_3)Q_{\mu\nu}Sp[\beta\gamma_5\Lambda_+(p_2)\beta\gamma_\nu\Lambda_+(p_4)]$

(5) $\bar{\psi}(p_3)\gamma_5\Lambda_+(p_1)\beta\gamma_\nu\Lambda_+(p_4)\beta\gamma_5\Lambda_+(p_2)\beta\gamma_\mu\psi(p_3)Q_{\mu\nu}$

(6) $\bar{\psi}(p_3)\gamma_\mu\Lambda_+(p_1)\beta\gamma_k\Lambda_+(p_4)\beta\gamma_\nu\Lambda_+(p_2)\beta\gamma_\lambda\psi(p_3)Q_{\mu\nu}Q'_{\lambda k}$

There will be two terms, each of the type (1), (2), (3) and (6) and four terms each of the type (4) and (5).

In the above expressions, while simplifying, we can very easily get rid of the γ_5's when they occur in pairs by transposing and putting $(\gamma_5)^2 = 1$, which introduces a change of sign at every step when a γ_5 crosses a γ_μ. It can be easily seen that the terms like (4) which do not contain γ_5 in suitable pair will vanish since the spur becomes identically zero.

To consider the matrix elements between $\bar{\psi}(p_3)$ and $\psi(p_3)$ in the different type of terms, we shall first prove the following result : When

$$\begin{bmatrix} a_1+i\vec{\sigma}\,.\,\vec{b}_1 & ia_2+\vec{\sigma}\,.\,\vec{b}_2 \\ ia_3+\vec{\sigma}\,.\,\vec{b}_3 & a_4+i\vec{\sigma}\,.\,\vec{b}_4 \end{bmatrix} \begin{bmatrix} a_1'+i\vec{\sigma}\,.\,\vec{b}_1' & ia_2'+\vec{\sigma}\,.\,\vec{b}_2' \\ ia_3'+\vec{\sigma}\,.\,\vec{b}_3' & a_4'+i\vec{\sigma}\,.\,\vec{b}_4' \end{bmatrix}$$
$$= \begin{bmatrix} A_1'+i\vec{\sigma}\,.\,\vec{B}_1' & iA_2'+i\vec{\sigma}\,.\,\vec{B}_2' \\ iA_3'+\vec{\sigma}\,.\,\vec{B}_3' & A_4'+i\vec{\sigma}\,.\,\vec{B}_4' \end{bmatrix} \qquad \dots \text{(A1)}$$

and if a_k, $\vec{b}_k$, a_k', $\vec{b}_k'$ $(k = 1, 2, 3, 4)$ are all real, then also A_k', $\vec{B}_k'$ $(k = 1, 2, 3, 4)$ are all real; *i.e.* the product of any two (and hence any number) of matrices of the above form (A1) remains unaltered in form with the real and imaginary parts remaining in tact. This result may be verified by direct multiplication.

It can very easily be checked that (1) does not contain any polarisation term. In the second, we find that once we get rid of the γ_5's the remaining matrices will be of the form (A1) and hence their products will also have the same form, and thus by (15*a*) will not give rise to any polarisation dependent scattering cross-section when we add to it the complex conjugate expression.

For considering the terms of type (3), we first consider the case that $Q_{\mu\nu}$ is expressed as direct product of two energy-momentum vectors. In this case, clearly the operator within $\bar{\psi}(p_3)$ and $\psi(p_3)$ is of the form (A1) as is easily seen by multiplying by $-i^2$ and noticing that $i\gamma_p$ for all values of p is of the form (A1). The spur in this term is easily seen to be real, and hence this term does not contribute anything to a polarisation dependant cross-section. In an exactly similar manner we can also see that when $Q_{\mu\nu}$ or $Q_{\mu\nu}'$ are (or as a corollary, sums of) momenta, none of the other types of terms can contribute anything to any dependance on polarisation.

Even when one (both) of $Q_{\mu\nu}$ and $Q_{\lambda k}'$ is replaced by $\delta_{\mu\nu}$ or (and) $\delta_{\lambda k}$ with other real coefficients, it can be seen in these terms that they will be sum of terms with all their matrices of the form (A1). This can be proved, for example, by separating a $\gamma_\mu \ldots \gamma_\mu$ summation into two parts, as $\gamma_k \ldots \gamma_k$ (with $k = 1, 2, 3$) and $\gamma_4 \ldots \gamma_4$. It will be seen that we shall either require two i's to make the matrices of the form (A1) and with real coefficients, or they will be automatically of that form.

As stated in the main text, this leads to the conclusion that the scattering cross-section will be independent of the polarisation state of $\psi(p_3)$.

ACKNOWLEDGMENT

The author wishes to express his sincere thanks to Prof. D. Basu, Ph. D. of the Indian Association for the Cultivation of Science for many fruitful discussions.

REFERENCES

Bhatia, A. B., 1950, *Proc. Phys. Soc. (Lond.)* **A63**, 502.
Heitler, W., 1954, The Quantum Theory of Radiation.
Lepore, J. V., 1950, *Phys. Rev.* **79**, 137.
Mott, N. F., 1932, *Proc. Roy. Soc.*, **A135**, 429.
Verde, M., 1955, *Nuvovo Cim.*, **12**, 452.